Void Fraction Distribution in Beds of Spheres

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The radial variation of void fraction within a packed bed of uniform spheres has been found to vary from unity at the wall to about 38% in the interior of large beds. The voidage distribution takes the form of a damped oscillatory wave with the oscillations damped out at about 4½ to 5 sphere diameters from the container wall. The distribution function is essentially independent of whether the container wall is concave or convex.

Continuous surfaces within the bed are responsible for local voidage variations similar to those at the container wall.

An improved experimental technique is described in detail. The results presented are for a number of cases with *D/d* varying from 2.6 to infinity, and for boundary walls which are concave and convex.

Knowledge of the void fraction distribution within a packed bed is essential for any rigorous analysis of the fluid dynamics within that bed. Several investigators (*1, 2, 3*) have measured velocity distribution within a packed bed, and some (*3, 4*) have measured heat and mass transfer coefficients. Attempts to correlate data of this type have been hampered by a lack of knowledge of the detailed internal composition of the packed bed which in turn influences the velocity perturbations within the bed.

Study of the geometry of stacked spheres indicates that unit cell voidages can vary from about 26 to almost 48%, depending on the particular stacking arrangement (*5*). Large randomly packed beds of uniform spheres tend to pack with an average void fraction of 39%. This is of course not the entire story, since locally the voidage varies from point to point. Near the wall of the containing vessel the void fraction will be larger than near the center of the bed, since at the wall the spheres must conform to the wall's curvature. Immediately adjacent to the wall the void fraction should approach unity, because each particle can make only point contact with the wall. In the center of the bed, that is, far removed from the wall, the stacking arrangement should not be influenced by the presence of the wall, and a minimum voidage should be observed.

Attempts have been made in the past to express the wall effect mathematically (*6, 7*), with generally unsatisfactory results, since the assumptions involved in developing the mathematical expression depart appreciably from the actual conditions of the system For example Leva (*7*) assumed constant porosity from a point 1 particle radius in from the wall to the center of the bed. Actually the perturbations in the voidage introduced by the presence of a retaining wall extend inward from the wall a distance of about 10 particle radii. Smith et al. (*2*) report the results of Shaffer (*8*) who attempted to experimentally determine the voidage distribution in a bed. Shaffer's (*8*) experimental technique consisted of the addition of known quantities of water to a packed bed lying on its side, that is axis of cylinder horizontal, and noting the change in height of the liquid with each additional increment. Obviously each increment involved a slab region of increasing chord length. To reduce these measurements to radial voidage distributions involved invalid assumptions and the conclusions suffered thereby.

Tierney et al. (*9*) reported radial void distributions obtained by the following technique. A packing material, such as cork spheres, Berl saddles, or Raschig rings, was poured into a cardboard cylinder which was then filled slowly with hot wax. After the wax solidified, the bed was sawed into circular slabs which were in turn sawed into concentric rings in specially designed jigs. Analysis for void fraction was made by first removing the wax from the packing material, either by melting or by dissolution of the wax in benzene, then distilling the benzene to recover the wax. The void volume was then calculated from the weight of wax recovered and its density.

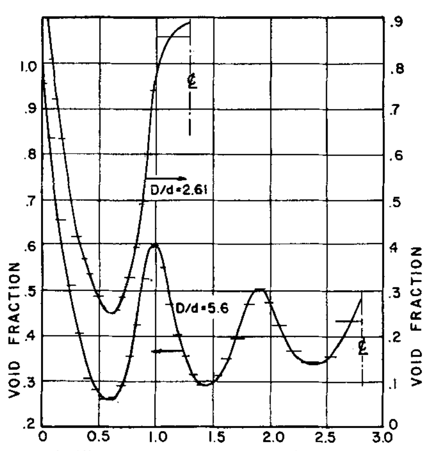

Fig. 1. Void fraction in beds of uniform spheres *D/d* = 2.61 and *D/d* = 5.6.

The results of Tierney et al. are in essential agreement with those presented here. However the major contributions of this paper over that of the work described above are: (1) the presentation of a technique which is at once more accurate, less tedious to apply, and easily adaptable to virtually any type of packing material and distribution study using only standard machinists' equipment; (2) the extension of data for spherical packings to a much larger range of *D/d* ratios; and (3) the investigation of the effect of convex surfaces (for example heat exchanger tubing) in a packed bed.

Kimura et al. (*10*) in a still older paper report the results of a technique which is basically similar to that of Tierney, but which are in disagreement with those of Tierney and the present authors. It is probable that Kimura's particles, having been produced by crushing and screening, were not true spheres nor of uniform size. This probably accounts for their failure to observe a cycling voidage.

EXPERIMENTAL TECHNIQUE

The experimental technique employed consisted of pouring uniformly sized spherical lead shot into a container and then filling all the interstices with a liquid epoxy resin. Upon curing of the resin the solid cylinder so obtained was machined, in stages, to successively smaller diameters. The weight and diameter of the cylinder were measured after each machining cut. In this manner the average density of each annular ring removed by the machining operation could be determined. By simple material balance this average density may be shown to be related to the average voidage of the part removed by the following formula:

$$\epsilon = V_p/V_m = \frac{\rho_L - \rho_m}{\rho_L - \rho_p} \qquad (1)$$

In preparing the test specimen lead shot was cleaned in a detergent solution to improve the adherence of the epoxy which was later poured into the container. No attempt was made to obtain any specified mode of packing of the shot into the con-

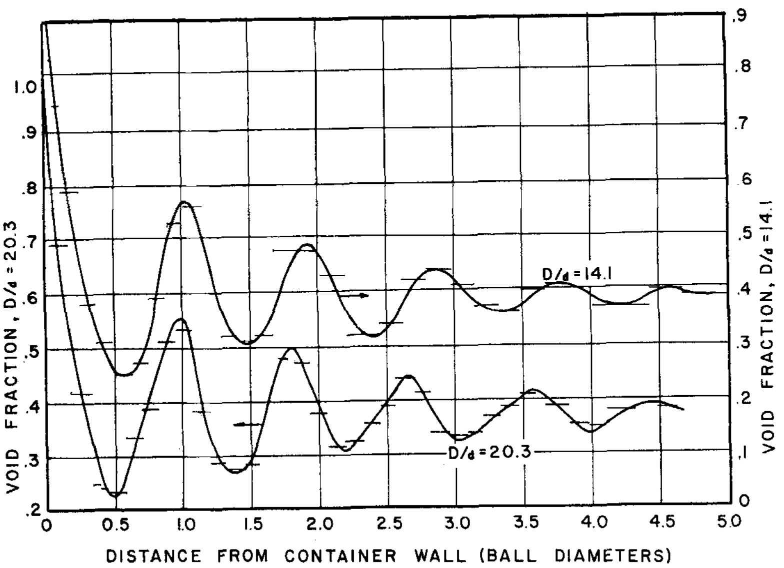

Fig. 2. Void fraction in beds of uniform spheres D/d = 14.1 and D/d = 20.3.

tainer, nor was the container vibrated in any way to compact the shot after pouring. The epoxy resin was introduced into the container from the bottom and flowed upward through the bed, and displaced air as it filled the bed. The resin flow rate was low enough that it did not disturb the packing as it filled the voids in the bed. The resin was allowed to cure for 96 hr. at room temperature, followed by 4 hr. at 125°F.

After it had been cured, the solidified packed bed was prepared for machining in a standard machinist's lathe. The ends were faced and the cylinder was turned down to progressively smaller diameters. The depth of each cut was adjusted so that a minimum of six cuts had to be taken for each ball diameter removed. Thus, for a bed of 0.08-in. particles, 0.013 in. was removed with each cut. However no cut deeper than 0.026 in. was taken, irrespective of the particle size.

The length and diameter were measured before and after each cut, with micrometers capable of being read to ± 0.0002 in. After each cut the piece was weighed with a pulp balance with a 5 mg. sensitivity.

The operation of alternately machining off part of the bed followed by weighing and remachining, etc., was continued until a bed diameter of about ¾ in. was reached. Any attempt to machine much further resulted in a broken bed. To determine the void fraction into the center of the bed the ¾-in. diam. bed was drilled axially with a series of concentric holes, starting with 1/16 in. and increasing the hole size in increments of 1/32 in. After each hole was drilled, the weight and volume of the bed was determined, the volume being determined by water displacement.

EXPERIMENTAL RESULTS

Table 1 lists the cases studied.

The data obtained in this series of experiments are indicated in Figures 1 through 5. In each case the voidage at the wall is 100%, and the voidage falls to a minimum value of about 25%, a distance of approximately ½ ball diameter away from the wall. The voidage then oscillates through several maxima and minima before settling out at the mean voidage of a random packed bed.

Owing to the method used to develop this data, each data point is necessarily an average voidage for a thin cylindrical element of the bed. Each data point is plotted as a short horizontal line, the width of which is equal to the cylindrical element machined off, while the ordinate value is equal to the average voidage.

It is apparent that it would be desirable to take as thin a section as possible in order to accurately define the voidage distribution curve. On the other hand it has been shown (*11*) that the accuracy of any single measurement is inversely proportional to the thickness of the cylindrical section removed. Thus it is necessary to strike a balance between the accuracy of a measurement and the precision of the voidage distribution curve resulting from these measurements.

The equation relating the standard deviation of a measurement and the measured parameters is

$$\sigma(\epsilon) = \frac{\rho_m}{\sqrt{2}(\rho_L - \rho_p)} \frac{\sigma_{(D)}}{t} \qquad (2)$$

The above equation showed that if at least six measurements were taken within a thickness of 1 ball diameter, the voidage distribution curve would be well defined, while at the same time the confidence band on a single measurement remained narrow. The average standard deviations of the void fraction for each run, as estimated by Equation (2) are given with the curves for Figures 3 and 4.

As is indicated in Table 1, duplicate runs were made on several beds. These runs were made in order to determine the reproducibility of a bed formed simply by pouring. The results of each set of duplicate runs were in excellent agreement. Only one run from each D/d ratio is shown in Figures 1 to 4. However the results of the duplicate runs may be found in reference *11*.

Study of the voidage variation from a flat plate required a slight modification in technique. A large diameter bed was prepared with a flat lucite plate at the bottom. After the resin was cured, the bed was removed from the mold and machined to remove the outer circumferential layer in which the radial fluctuations in voidage were found to exist. Actually a thickness of about 7 ball diameters was machined away, which is somewhat more than was required to eliminate radial variations. Subsequent machining operations

TABLE 1

Case	Ball diam., in., d	D/d	Number of runs
1	0.622	∞	1
2	0.080	20.3	2
3	0.115	14.1	2
4	0.290	5.6	2
5	0.622	2.61	2

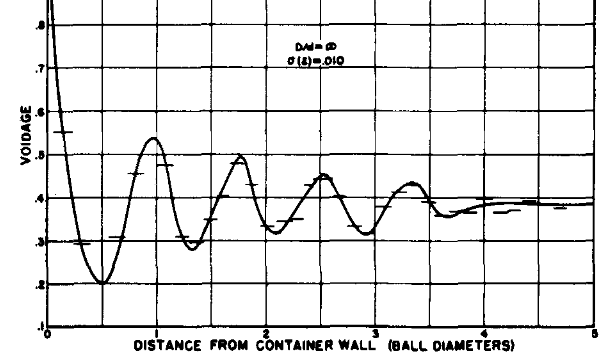

Fig. 3. Void fraction in beds of uniform spheres D/d = ∞.

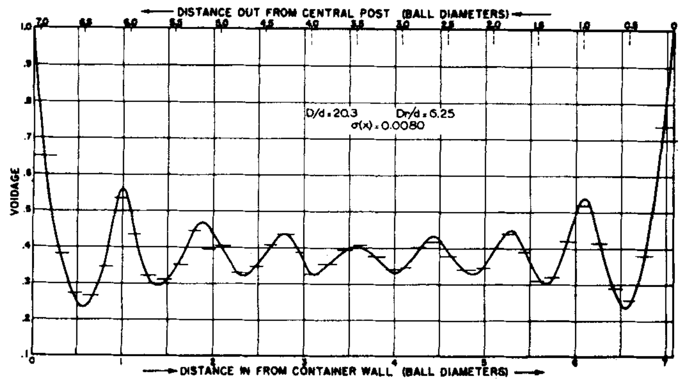

Fig. 4. Void fraction in a large bed of uniform spheres containing a central post.

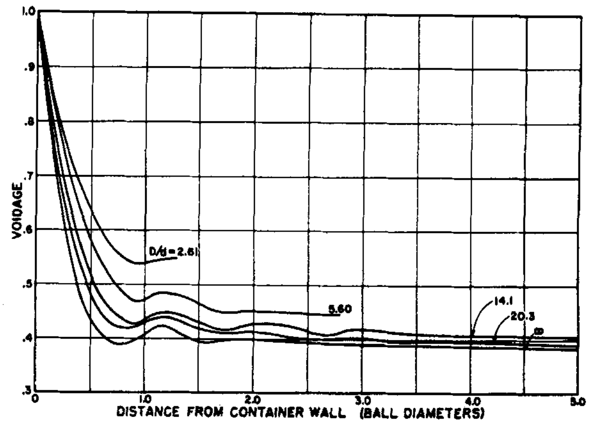

Fig. 5. Integrated void fractions in beds of uniform spheres for various *D/d* ratios.

consisted of taking a series of facing cuts, starting at the bottom face and working progressively upward. The data for this series of runs is shown in Figure 3. It will be noted that the same pattern of oscillations is observed and that they extend into the bed a distance of 4½ ball diameters.

One last series of runs was made to determine the effect which internal structures (such as heat exchanger tubes in a packed bed) would have on the void distribution in the bed. To study this effect a bed was prepared by installing a steel rod on the axial center line of the mold, and then pouring and preparing the bed in the usual way. The outer surface was machined inward to the steel rod which was mounted between lathe centers. Results are shown in Figure 4. Note that the same pattern of oscillations exists regardless of whether the surface bounding the bed is concave (vessel wall) or convex (internal tube). It would appear that a distance of 4½ ball diameters is required from concave, flat, or convex surfaces before wall effects are damped out. In the case of the last series of experiments the total distance between the ball and the rod was only 7.2 ball diameters; hence the wall effect had not yet damped out before the rod effect began to take over.

The average voidage of a randomly packed bed of uniform spheres is usually quoted as 39%. By integrating the local voidage with volume, the average voidage from the wall to any point within the bed can be found. The average voidage so obtained is shown in Figure 5. It will be seen that the average voidage approaches 39%, but for large diameter beds only.

It is interesting to compare these results with a hindsight analysis of a packed bed. Consider a two-dimensional representation of how spheres tend to fill a cylindrical vessel. The vessel wall influences the orientation of the outermost spheres, forcing a row of spheres to form along the walls. Since these spheres have only point contact with the containing cylinder, the voidage at the wall must be unity. Proceeding in from the wall at the voidage must decrease to a minimum (on a line drawn through the contact points of this outer row of spheres), after which it would start to increase. It cannot increase to unity, however, since the second row of spheres rests in the cusps formed by the spheres in the first row. Proceeding in from the wall, the pattern is repeated and, since each row is more random than the row which precedes it, the voidage oscillates around the mean value with a decreasing amplitude, which is not completely damped out until a point is reached about 5 ball diameters in from the wall.

Since the row adjacent to the wall is so highly oriented, it is possible to calculate where the minimum voidage should be located. This is given by

$$\delta = 1/2 + (D/d - 1) - \sqrt{(D/d - 1)^2 - 1} \qquad (3)$$

The agreement obtained between Equation (3) and the experimental data is indicated in Table 2.

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NOTATION

D = bed diameter
d = ball diameter

TABLE 2

D/d	$(\delta - ½)$ calc	$(\delta - ½)$ obs
2.61	0.173	0.16
5.60	0.055	0.06
14.1	0.027	0.03
20.3	0.019	0.0
∞	0.0	0.0
6.25	0.035	0.05

ϵ = voidage
V_p = volume of plastic in the part removed
V_m = total volume removed

Greek Letters

ρ_L = density of lead shot
ρ_m = average density of the part removed
ρ_p = density of cured epoxy resin
$\sigma_{(\epsilon)}$ = standard deviation of the void fraction
$\sigma_{(D)}$ = standard deviation of the diameter measurement
δ = distance from the wall (expressed in ball diameters) at which the minimum voidage is located

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